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CONCERNING MULTIPLE INTERPRETATIONS OF POSTULATE SYSTEMS AND THE "EXISTENCE" OF HYPERSPACE

WHAT do we mean when we speak of n-dimensional space and n-dimensional geometry, where n is greater than 3? The question refers to talk about space and geometry that are n-dimensional in *points*, for ordinary space, as is well known, is 4-dimensional in lines, 4-dimensional in spheres, 5-dimensional in flat line-pencils, 6-dimensional in circles, etc., and there is naturally no mystery involved in speaking of these latter varieties of multi-dimensional manifolds and their geometries, no matter how high the dimensionality may be. No mystery for the reason that in these geometries everything lies within the domain of intuition in the same sense in which everything in ordinary (point) geometry lies in that domain. In other words, these n-dimensional geometries are nothing but theories or geometries of *ordinary* space, that arise when we take for element, not the point, but some other entity, as the line or the sphere, . . ., whose determination in ordinary space requires more than 3 independent data. Of these varieties of n-dimensional geometry, the inventor was Julius Plücker (d. 1868), but Plücker declined to concern himself with spaces and geometries of more than four dimensions in *points*.

Since Plücker's time, however, such hyper-theories of points have invaded not only almost every branch of pure mathematics, but also —strangely enough—certain branches of physical science, as, for example, the kinetic theory of gases. As to the manner of this latter invasion a hint may be instructive. Given N gas molecules enclosed, say, in a sphere. These molecules are, it is supposed, flying about hither and thither, all of them in motion. Each of them depends on six coordinates, x, y, z, u, v, w, where x, y, z are the usual positional coordinates of the molecule regarded as a point in ordinary space, and u, v, w are the components of the molecule's velocity along the three coordinate axes. Knowing the six things about a given mole-

cule, we know where it is and the direction and rate of its going. The N molecules making up the gas depend on $6N$ coordinates. At any instant these have definite values. These values together define the "state" of the gas at that instant. Now these $6N$ values are said to determine a point in space of $6N$ dimensions. Thus is set up a one-one correspondence between such points and the varying gas states. As the state of the gas changes, the corresponding point generates a locus in the space of $6N$ dimensions. In this way the behavior or history of the gas gets geometrically represented by loci in the hyperspace in question.

Is such geometric n-dimensional phraseology merely a geometric way of speaking about non-spatial things? Even if there exists a space, S_n, one *may* employ the language appropriate to the geometry of the space without having the slightest reference to it, and, indeed, without knowing or even enquiring whether it exists. This use of geometric speech in discourse about non-spatial things is not only possible, but in fact very common. An easily accessible example of it may be found in Bôcher[1] where, in speaking of a set of values of n independent variables as a point in space of n dimensions, the reader is told that the author's use of geometric language for the expression of algebraic facts is due to certain advantages of that language compared with the language of algebra or of analysis; he is told that the geometric terms will be employed "in a wholly conventional algebraic sense" and that "we do not propose even to raise the question whether in any geometric sense there is such a thing as space of more than three dimensions."

It is held by many, including perhaps the majority of mathematicians, that there are no hyperspaces of points and that n-dimensional geometries are, rightly speaking, not geometries at all, but that the facts dealt with in such so-called geometries are nothing but algebraic or analytic or numeric facts expressed in geometric language. If this opinion be correct, then the extensive and growing application of geometric language to analytical theories of higher dimensionality indicates a high superiority of geometric over analytic speech, and it becomes a problem for psychology to ascertain whether the mentioned superiority is adequate to explain the phenomenon in question and, if it be adequate, to show wherein the superiority resides.

No doubt geometric language has a kind of esthetic value that is lacking in the speech of analysis, for the former, being transfused with the rich reminiscences of sensibility, constantly awakens a delightful sense, as thinking proceeds, of the colors, forms, and motions of the sensuous world. This is an emotional value. No doubt, too,

[1] "Introduction to Higher Algebra," page 9.

geometric language has, in its distinctive conciseness, an economic superiority, as when, for example, one speaks of the points of the 4-dimensional sphere, $x^2+y^2+z^2+w^2=r^2$, instead of speaking of the various systems of values of the variables x, y, z, w that satisfy the equation $x^2+\cdots=r^2$. Additional advantages of geometric over analytic speech are brought to light in the following remarks by Poincaré in his address, "L'Avenir des Mathématiques" (1908):

"Un grand avantage de la géométrie, c'est precisément que les sens y peuvent venir au secours de l'intelligence, et aident à deviner la route à suivre, et bien des esprits préfèrent ramener les problèmes d'analyse à la forme géométrique. Malheureusement nos sens ne peuvent nous mener bien loin, et ils nous faussent compagnie dès que nous voulons nous envoler en dehors des trois dimensions classiques. Est-ce à dire que, sortis de ce domaine restreint où ils semblent vouloir nous enfermer, nous ne devons plus compter que sur l'analyse pure et que toute géométrie à plus de trois dimensions est vaine et sans objet? Dans la génération qui nous a précédés, les plus grands maîtres auraient répondu 'oui'; nous sommes anjourd'hui tellement familiarisés avec cette notion que nous pouvons en parler, même dans un cours d'université, sans provoquer trop d'étonnement.

"Mais à quoi peut-elle servir? Il est aisé de le voir: elle nous donne d'abord un langage très commode, qui exprime en termes très concis ce que le langage analytique ordinaire dirait en phrases prolixes. De plus, ce langage nous fait nommer du même nom ce qui se ressemble et affirme des analogies qu'il ne nous permet plus d'oublier. Il nous permet donc encore de nous diriger dans cet espace qui est trop grand pour nous et que nous ne pouvons voir, en nous rappelant sans cesse l'espace visible qui n'en est qu'une image imparfaite sans doute, mais que en est encore une image. Ici encore, comme dans tous les exemples précédents, c'est l'analogie avec ce qui est simple qui nous permet de comprendre ce qui est complexe."

The question of determining the comparative advantages and disadvantages of the languages of geometry and analysis is a very difficult one. It is evidently in the main a psychological problem. It appears that no serious and systematic attempt has ever been made to solve it. Here, it seems, is an inviting opportunity for a properly qualified psychologist, it being understood that proper qualification would include a familiar knowledge of the languages in question. The interest and manifold utility of such a study are obvious. In the course of such an investigation it would probably be found that the superiority of geometric over analytic speech is alone sufficient to account for the extensive and rapidly increasing literature of what is called n-dimensional geometry and that, in order to account for the

rise of such literature, it is therefore not necessary to suppose the existence of n-dimensional spaces, S_n, the facts dealt with in the literature being, it could be supposed, nothing but analytic facts expressed in geometric language.

If such a result were found, would it follow that S_n does not exist and that consequently n-dimensional geometry must be nothing but analysis in geometric garb? The answer is, no; for we may and we often do assign an adequate cause of a phenomenon or event without assigning the actual cause; and so the possibility would remain that n-dimensional geometry has an appropriate object or subject, namely, a space S_n which, though without sensuous existence, yet has every kind of existence that may warrantably be attributed to ordinary geometric space, S_3. For this last, though it is imitated by (or imitates) sensible space, as an ideal model or pattern is imitated by (or imitates) an imperfect copy, it is not identical with it. S_3 is not tactile space, nor visual space, nor that of muscular sensation, nor the space of any other sense, nor of all the senses—it is a conceptual space; and whether there are or are not spaces S_4, S_5, etc., which have every sort of existence rightly attributable to ordinary geometric space, S_3, and which differ from the latter only in the accident of dimensionality and in the further accident that S_3 appears in the rôle of an ideal prototype for an actual sensible space, whilst S_4, S_5, etc., do not present such an appearance,—that is the question which remains for consideration.

A friend called at my study, and, finding me at work, asked, "What are you doing?" My reply was: "I am trying to tell how a world which probably does not exist would look if it did." I had been at work on a chapter of what is called 4-dimensional geometry. The incident occurred ten years ago. The reply to my friend no longer represents my conviction. Subsequent reflection has convinced me that a space, S_n, of four or more dimensions has every kind of existence that may be rightly ascribed to the space, S_3, of ordinary geometry.

The following paragraphs present—merely in outline, for space is lacking for a minute presentation—the considerations that have led me to the conclusion above stated.

Let sensible space be denoted by sS_3. We know that sS_3 is discontinuous (in the mathematical sense of the term) and that it is irrational. By saying that it is irrational I mean what common experience as well as the results of experimental psychology prove: that three sensible extensions of a same type, let us for definiteness say three sensible lengths, l_1, l_2, l_3, may be such that

$$(1) \qquad l_1 = l_2,\ l_2 = l_3,\ l_1 \neq l_3.$$

Because sS_3 is thus irrational, because it is radically infected with such contradictions as (1), this space is not, and can not be, the subject or object of geometry, for geometry is rational: it does not admit three such extensions as those in (1). Not only do such contradictions as (1) render sS_3 impossible as a subject or object of geometry, but, when encountered, they produce intolerable intellectual pain—nay, if they could not in somewise be transcended or overcome, they would produce intellectual death, for, unless the law of non-contradiction be preserved, concatenative thinking, the life of intellect, must cease. In case of intellect we may say that its struggle for existence is a struggle against contradictions. But mere existence is not the characteristic aim or aspiration of intellect. Its aim, its aspiration, its joy, is compatibility. Indeed, intellect seems to be controlled by two forces, a *vis a tergo* and a *vis a fronte:* it is driven by discord and drawn by concord. Intellect is a perpetual suitor, the object of the suit being harmony, the beautiful daughter of the muses. Its perpetual enemy is the immortal demon of discord, ever being overcome, but never vanquished.

The victory of intellect over the characteristic contradictions inherent in sS_3 is won through what we call conception. That is to say that either we find or else we create another kind of space which, in order to distinguish it nominally and symbolically from sS_3, we may call conceptual space, and denote by cS_3. Unlike sS_3, cS_3 is mathematically continuous and it is rational. Like sS_3, cS_3 is extended, it has room, but the room and the extensions are not sensible, they are conceptual; and these extensions are such that, if l_1, l_2, l_3 be three amounts of a given type of extension, as length, say, and if $l_1 = l_2$ and $l_2 = l_3$, then $l_1 = l_3$. The space cS_3, whether we regard it as found by the intellect or as created by it, is the subject or object of geometry. The current vulgar confusion of sS_3 and cS_3 is doubtless due to the fact that the former imitates the latter, or the latter the former, as a sensible thing imitates its ideal, or as an ideal (of a sensible thing) may be said to imitate that thing; for it is precisely such alternative or mutual imitation that enables us in a measure to control the sensible world through its conceptual counterpart; and so the exigencies of practical affairs and the fact that reciprocally imitating things each reminds us of the other cooperate to cause the sensible and the ideal, the perceptual and the conceptual, to mingle constantly and to become confused in that part of our mental life that belongs to the sensible and the conceptual worlds of three dimensions. Nevertheless, it is a fact to be borne in mind that cS_3 is a subject or object of geometry and that sS_3 is not.

Now, in order to construct the geometry in question, we start with a suitable system of postulates or axioms expressing certain rela-

tions among what are called the elements of cS_3. These postulates, together with such propositions as are deducible from them, constitute the geometry of cS_3. I shall call it *pure* geometry, for a reason to be given later, and shall denote it by pG_3. For definiteness let us refer to the famous and familiar postulates of Hilbert. Any other system would do as well. In the Hilbert system, the elements are called points, lines, and planes. It is customary and just to point out that the terms point, line, and plane are not defined, and in critical commentary it is customary to add:

(*A*) That, consequently, these terms may be taken to be the names of any things whatsoever with the single restriction that the things must satisfy the relations stated by the postulates;

(*B*) That, when some admissible or possible interpretation I has been given to the element-names, the postulates P together with their deducible consequences C constitute a definite theory or doctrine D;

(*C*) That replacing I by a different interpretation I' produces no change whatever in D;

(*D*) That this invariant D is Euclidean geometry of three dimensions; and

(*E*) That, if we are to speak of D as a theory or geometry of a space, this space is nothing but the ensemble of any kind of things that may serve for an interpretation of P.

That the view expressed in that so-called "critical commentary" does not agree with common sense or with traditional usage is obvious. That it will not bear critical reflection can, I believe, be made evident. Let us examine it a little. In order to avoid the prejudicial associations of the terms point, line, and plane, we may replace them by the terms "roint," "rine," and "rane," so that the first postulate, or axiom, as Hilbert calls it, will read: *Two distinct roints always completely determine a rine.* Or, better still, we may replace them by the symbols e_1, e_2, e_3, so that the reading will be: *Two distinct e_1's always completely determine an e_2*; and similarly for the remaining postulates.

We will suppose the phrasing of (*A*), (*B*), (*C*), (*D*), (*E*), slightly changed to agree with the indicated new phrasing of the postulates.

It seems very probable that there are no termless relations, *i. e.*, relations that do not relate. It seems very probable that a relation to be a relation must be something actually connecting or subsisting between at least two things or terms. A postulate expressing a relation having terms is at all events ostensibly a statement about the terms, and so it would seem that, if the relation be supposed to be termless, the statement ceases to be a statement about something and, in so ceasing, ceases to be a statement that is true or else is false. In

discourse, it is true, there is frequent seeming evidence that relations are often thought of as termless, as when, for example, we speak of "a relation *and* its terms"; but then we speak also of a neckless fiddle without intending to imply by such locution that there can be a fiddle without a neck. As, however, we do not wish the validity of the following criticism to depend on the denial of the possibility of termless relations, the discussion will be conducted in turn under each of the alternative hypotheses: (h_1) There are termless relations; (h_2) There are no termless relations. We will begin with

HYPOTHESIS h_2

To (A) we make no objection.

Let us now suppose given to P some definite interpretation I. Let us grant that we now have a definite doctrine D, consisting of P and C. Either the things which in I the e's denote have or they have not content, character, or meaning, m, in excess of the fact that they satisfy P.

(1) Suppose they have not an excessive meaning m. Denote the interpretation by I_1 and the doctrine by D_1. This D_1 is a queer doctrine. We may ask: what does D_1 relate or refer to? That is, what is it a doctrine of or about? The question seems to admit of no intelligent or intelligible answer. For if the doctrine is about something, it is, it seems natural to say, a doctrine about the I_1-things (denoted by the e's); but, by (1), these I_1-things can not be characterized or indicated otherwise than by the fact of their satisfying P; and so it appears that such attempted natural answer is reducible and equivalent to saying (a) that the doctrine D_1 is about the things which it is about. In order not to be thus defeated, one might try to give an informing answer by saying that D_1 is a doctrine, not about the I_1-things, *i. e.*, not about terms of relations, but about the relations themselves. Such an answer is suspicious on account of its unnaturalness, and it is unnatural because the propositions of D_1 wear the appearance of talking explicitly, not about relations, but about terms of relations. Moreover, the answer is not an informing one unless the relations that the doctrine D_1 is alleged to be about can be characterized otherwise than by the fact of their being satisfied by the I_1-things, for, if they can not be otherwise characterized, evidently by (1) the answer reduces to a form essentially like that of (a). May not one escape by saying that the relations which D_1 is alleged to be a doctrine about are just the relations expressed by the propositions in D_1? Does this attempted characterization make the answer in question an informing one? If D_1 is a doctrine about the relations expressed by its propositions, then D_1 says or teaches some-

thing about these relations, for every doctrine, if it be about something, must teach or say something about that which it is about. In the case supposed, what does D_1 teach about the relations? Nothing except that they are satisfied by the I_1-things. In other words, what D_1 teaches about the relations expressed by its propositions is, by (1), that these are satisfied by things that satisfy them—a not very nutritious lesson. It is possible to make a yet further attempt so to indicate the relations as to render the answer, that the doctrine D_1 is about relations, an informing one. It is known that P may receive an interpretation I' different from I_1 in that the I'-things do not satisfy (1), but have an excessive content, character, or meaning m. May we not give the required indication of the relations that D_1 is said to be a doctrine about by saying that they are relations satisfied by the I'-things, the presence of the m involved making the indication genuine or effective? It seems so at first. But if again we ask what D_1 teaches about the relations thus indicated, we are led into the same difficulty as above. Moreover, when we ask what D_1 is a doctrine about we expect an answer in terms in somewise mentioned or intimated in the D_1 discourse, whilst in the case in hand the required indication has depended on m, a thing expressly excluded from the D_1 discourse by (1).

So, I repeat, D_1 is a queer doctrine.

It must be aded that if there be an interpretation I_1, it is unique of its kind; for if I_1' were an interpretation satisfying (1), the I_1'-things would have no excesive meaning m; hence they would be simply the I_1-things, and I_1 and I_1' would be merely two symbols for a same interpretation.

Accordingly, if there were an interpretation I_1, but no other, *i. e.*, no interpretation I in which the I-things did not satisfy (1), then (C) would be pointless; by (D), D_1 would be Euclidean geometry of three dimensions; and, by (E), Euclidean space, if we wished to speak of D_1 as a geometry of a space, would be the ensemble of the I_1-things; but, if we wished to characterize the I_1-things, the elements of Euclidean space, we could only say that they are the things satisfying certain relations, and, if we wished to indicate what relations, we could only say, the relations satisfied by those things: a very handsome circle.

In the following it will be seen that we are in fact not imprisoned within that circle.

(2) Suppose the I-things of the above-assumed interpretation I do not satisfy (1), but have an excessive meaning m. (It is known that such an I is possible, an example being found by taking for an e_1 any ordered triad of real numbers (x, y, z); for an e_2 the ensemble

of triads satisfying any two distinct equations,

$$A_1x + B_1y + C_1z + D_1 = 0, \quad A_2x + B_2y + C_2z + D_2 = 0,$$

in neither of which the coefficients are all of them zero; and for an e_3 the ensemble of triads satisfying any one such equation; the presence of m being evident in countless facts such as the fact, for example, that an e_1 is composed of numbers studied by school-boys or useful in trade without regard to their ordered triadic relationship.) Denote the assumed definite interpretation I by I_2 to remind us that it satisfies (2), and denote the corresponding doctrine by D_2. It is immediately evident that there is an interpretation I_1 and hence a doctrine D_1, for to obtain I_1 it is sufficient to abstract from the m of the I_2-things and to take the abstracts (which plainly satisfy (1)) for I_1-things.

Are D_1 and D_2 but two different symbols for one and the same doctrine, as asserted by (C)? Evidently not. For, in respect of D_2, we can give an informing answer to the question, what is D_2 a doctrine about? Owing to the presence of the m in the I_2-things, the answer will be an informing one whether it be the natural answer that D_2 is a doctrine about the I_2-things, or one of the less natural answers, that D_2 is about the relations having the I_2-things for terms, that D_2 is about the relations expressed by its propositions; whilst, as we have seen, owing to the absence of m in the I_1-things no such answers were, in respect of D_1, informing answers.

Can not (C) be saved by refusing to admit that there is an interpretation I_1, and so refusing to admit that there is a D_1? If there is no I_1 and hence no D_1, then (C) is pointless unless there is an I_2' and so a D_2' in which the I_2'-things have an m' different from the m of the I_2-things. But if there is an I_2' thus different from I_2, then obviously D_2' and D_2 are, contrary to (C), different doctrines, for they are respectively doctrines about the I_2-things and the I_2'-things, and these thing-systems are different by virtue of the difference of m and m'. Now, it is known that there are two such differing interpretations I_2 and I_2'. For we may suppose I_2 to be the possible interpretation indicated in the above parenthesis. And for I_2' we may take for e_1 any ordered triad of real numbers, except a specified triad (a, b, c), and including (∞, ∞, ∞); for e_2 the ensemble of triads, except (a, b, c), that satisfy any pair of equations,

$$\begin{aligned} A_1(x^2 + y^2 + z^2) + 2B_1(x - a) + 2C_1(y - b) \\ + 2D_1(z - c) - A_1(a^2 + b^2 + c^2) = 0, \\ A_2(x^2 + y^2 + z^2) + 2B_2(x - a) + 2C_2(y - b) \\ + 2D_2(z - c) - A_2(a^2 + b^2 + c^2) = 0; \end{aligned}$$

and for e_3 the ensemble satisfying any one such equation. Just as

when we compared D_1 and D_2, so here the conclusion is, that (C) is not valid.

As a matter of fact mathematicians know that there are possible infinitely many different interpretations of P. It follows from the foregoing that there are correspondingly many different doctrines. For the sake of completeness we may include D_1 among these, although, for the purpose of answering a hypothetical objection, we momentarily supposed D_1 to be disputable or inadmissible.

Which one of the D's is (or should be called) Euclidean geometry of three dimensions? I say which "one"? For, as no two are identical, it would be willful courting of ambiguity to allow that two or more of them should be so denominated. Which *one*, then? Evidently not one of the numerical ones, such, for example, as the two above specified. For who has ever *really* believed that a point, for example, *is* a triad of numbers? We know that the Greeks did arrive at geometry; we know that they did not arrive at it through numbers; and we know that, in their thought, points were not number triads, nor were planes and lines, for them, certain ensembles of such triads. The confusion, if anybody ever was really thus confused, is due to the *modern* discovery that number triads and certain ensembles of them happen to satisfy the same relations as the Greeks found to be satisfied by what they called points, lines, and planes. There is really no excuse for the confusion, for, if Smith is taller than Brown, and yonder oak is taller than yonder beech, it obviously does not follow that Smith is the oak and Brown the beech.

Evidently Euclidean geometry of three dimensions is that particular D for which the I-things are points, lines, and planes. Here it is certain to be asked: What, then, are points, lines, and planes? And the asker will mean to imply that, in order to maintain the proposition, it is necessary to define these terms. The proper reply is that it is not necessary to *define* them. All that can be reasonably required is that they be indicated, pointed out, sufficiently described for purposes of recognition, for what we desire is to be able to say or to recognize what Euclidean geometry is about. To the question one might, not foolishly, reply that the terms in question denote things that you and I, if we have been disciplined in geometry, converse understandingly about when we converse about geometry, though neither of us is able to say with *absolute precision* what the terms mean. For who does not know that it is possible to write an intelligent and intelligible discourse about cats, for example, without being able to tell (for who can tell?) precisely what a cat is? And if it be asked what the discourse is about, who does not know that it is an informing answer to say that it is about cats. It is informing because the term cat has an excessive meaning, a meaning

beyond that of satisfying the propositions (or relations) of the discourse.

Just here it is well worth while to point out an important lesson in the procedure of Euclid. Against Euclid it is often held as a reproach that he attempted to define the element-names, point, line, and plane, since no definitions of them could render any logical service, that is, in the strictly deductive part of the discourse. But to render no logical service is not to render no service. And the lesson is that the definitions in question, which it were perhaps better to call descriptions, do render an extralogical service. They render such service not only in guiding the imagination in the matter of invention, but also in serving to indicate, with a goodly degree of success, the excessive meaning m of the elements denoted by the terms in question and in thus serving to make known what it is that the deductive part of the discourse is about. One should not forget that no discourse, no doctrine, not even so-called pure logic itself, is exclusively deductive, for in any doctrine there is reference, implicit or explicit, to something extradeductive or extralogical, reference, that is, to something which the doctrine is about.

Are the three Euclidean "definitions," thus viewed as descriptions, sufficient or adequate to the service that they are here viewed as rendering? If by sufficient or adequate be meant exhaustive, the answer is, of course, no. For we may confidently say that no possible description, that is, no description involving only a finite number of words, possibly can exhaust the meaning of a system of terms except, possibly, in the special case where these have no meaning beyond what they must have in order merely to satisfy a finite number of postulates. But exhaustive is not what is meant by adequate. To employ a previous illustration, it is not necessary to give or to attempt an exhaustive description of "cat" in order to tell adequately what it is that a discourse ostensibly about cats is ostensibly about. It is a question of intent. A description is nearly, if not quite, adequate if it enables us to avoid thinking that terms are intended to denote what they are not intended to denote. And, whilst we may not admit that the three Euclidean "descriptions" are the best that can be invented for the purpose, yet we must allow that they have long served the end in question pretty effectively and that they are qualified to continue such service. They have been and they are good enough, for example, to save us from thinking that the things which in geometry have been denoted by the terms, points, lines, and planes, are identical with number triads, etc. The open secret of their thus saving us is no doubt in their causing us to think of points, lines, and planes in terms of, or in essential connection with, what we know as *extension,* whilst numbers and number

ensembles are not things naturally so conceived. For evidently the notions of "length" and "breadth" involved in the Euclidean "descriptions" are not *metric* in meaning; they do not signify definite or numeric quantities or amounts of something (as when we say the length of this or that thing is so and so much); but plainly they are generic notions connoting *extension.* It is safe to say that a mind devoid of the concept or the sense of extension could not know what things the "descriptions" aim at describing. It is true that Euclid's "description" of a point as "that which has no part" implies a *denial* of extension, but the denial is one of *extension,* and, in its contextual atmosphere, it is *felt* to be essential to an adequate indication of what is meant by point. On the other hand, if one were (and how unnatural it would be!) to describe an ordered triad of numbers as "that which has no part," it would be immediately necessary to explain away the seeming falsity of the description by saying that the triad is not the ordered multiplicity (of three numbers) *as* a multiplicity, but is merely the *uniphase* of the multiplicity, and that it is this uniphase which has no part. If, next, we were to say that thus extension is denied to the uniphase, the statement, though true, would be *felt* to be inessential to an adequate indication of what is meant by a triad of numbers. Such felt difference is alone sufficient to make any one pause who is disposed to adopt the current creed that a point is nothing but an ordered triad of numbers. It is not contended that a point is *composed* of extension; the contention is that point and extension are so connected that a mind devoid of the latter notion would be devoid of the former, just as a mind devoid of the notion of variable or variation would be devoid of the notion of constant, though a constant is not a thing *consisting* of variation; just as the notion of limit would not be intelligible except for the notion of something that may have a limit, though the limit is not *composed* of it; and just as an instant, which is not *composed* of time, would not be intelligible except for the notion of time.

In a discussion of such matters it is foolish and futile to talk about "proofs." The question, as said, is one of intent; it is a question of self-veracity, of getting aware of and owning what it is that we mean by the terms and symbols of our discourse. If, despite the Euclidean "descriptions" and despite any and all others that may supplement or supplace them, one fails to see that *extension* is essentially involved in the meaning that the terms points, lines, and planes, are intended to have, the failure will be because "as the eyes of bats are to the blaze of day, so is the reason in our soul to the things which are by nature most evident of all." Nothing is more evident than that there is something that is called extension. We have but to open our eyes to get aware that we are behold-

ing an *expanse*, something extended. We see things *as* extended; things *as* extended are revealed to the tactile sense; a region or room involving extension is a datum of the muscular sensations connected with our bodily movements; and so on. So much is certain. But it is said and rightly said that these are *sensible* things; that the extension they are revealed as having is *sensible* extension; that these sensibles are infected with contradiction, above noted, revealed in common experience, and confirmed by the psychophysical law of Weber and Fechner; that geometry is free from contradiction; that, therefore, geometry is not a doctrine about these sensibles; that among these sensibles are not the things which in geometry are denoted by the terms point, line, and plane; and that, if these terms imply or connote extension, as asserted, this extension is not sensible extension. Granted. The "connoted extension" is not sensible, it is conceptual. How know, however, that there is conceptual extension? The answer is, by arriving at it. (We need not here debate whether such "arriving" is best called creating or is best called finding.) But how does the mind arrive at it? By doing certain things to the sensibles, the raw material of mental architecture. What things? An exhaustive answer is unnecessary—perhaps impossible. The things are of two sorts: the mind gives to the sensibles; it takes away from them. Consider, for example, a sensible line. From it the conceptualizing intellect takes away (abstracts from, disregards) certain things that the sensible in question has or may have, as color, weight, temperature, etc., including *part* of the extension, thus, I mean, narrowing and thinning away all breadth and thickness. What of the extension called length? Have the narrowing and thinning taken it away? It was not so intended, the opposite was intended. Yet no sensible length (extension) remains. Does the narrowing and thinning involve shortening? We are absolutely certain that it does not. What, then, is it that has happened? Evidently that, by the indicated taking away, the mind has arrived at insensible length, one kind of insensible extension, that is, at conceptual length, one kind of conceptual extension. A stretch, we are sure, remains, but it is not a sensible stretch. The extension thus arrived at is yet not the extension connoted by or involved in the things that geometry is about, for in the taking-away process of arriving at it there is nothing to disinfect it of the contradictions inherent in the sensible with which we started. It remains, then, to follow the indicated process of taking away by a process of giving, that is to say, it remains to endow the conceptual extension (arrived at) with continuity so as to render it free from the mentioned contradictions. This done, the kind of extension meant in ordinary geom-

etry or ordinary geometric space is arrived at. Such is, in kind, the conceptual extension that, it is here held, is essential to what the geometric terms, point, line, plane, are intended to mean. Without further talk we may say that such extension is essential in the conceptual space that, we may say, ordinary Euclidean geometry is about in being about the elements of the space.

If we denote this conceptual space by cS_3 to distinguish it from (non-geometrizable) sensible space sS_3, then the geometry of cS_3, if constructed by means of postulates P making no indispensable use of algebraic analysis, may be called pure geometry, pG_3. If, as in the Caretesian method, we use ordered number triads, etc., as we may use them, not to be points, etc., but to represent points, etc., then we get analytical geometry, aG_3, of cS_3. On the other hand, if, as we may, we interpret the P by allowing the I-things to be number triads, etc., as above indicated, the resulting doctrine is, not geometry, but a pure algebra or analysis, pA_3. If we use points, etc., not to be, but to represent, number triads, etc., and so employ geometric language in constructing pA_3, we get by this kind of anti-Cartesian procedure, not a geometry, but geometrical analysis, gA_3.

Hypothesis h_1

It is unnecessary to say anything and is not worth while to say much under this hypothesis. For if the e's in P do not denote something, then as the relations (if there be any) are termless, the doctrine D (if there be one) is not about anything, unless about the relations, but about these it says nothing, for, if it says aught about them, what it says is that they are satisfied by certain terms whose presence in the discourse is excluded by h_1. We may profitably say, however, that, in the case supposed where the e's do not denote something but are merely uninterpreted variables ready, so to speak, to denote something—in this case we may say that, though there is no doctrine D, there is a *doctrinal function,* $\Delta(e_1, e_2, e_3)$. Then we should add that the doctrines that do arise from actualized possible interpretations of the e's are so many *values* of Δ. This function Δ, if we give some warning mark as Δ' to its symbol, may be further conveniently employed in talking about an ambiguous one of the doctrines in question, *i. e.,* about "any value," an ambiguous value, of the function. As above argued, these values, these doctrines are identical in *form,* they are *isomorphic,* all of them having the form of Δ, but no two of them are the same in respect of content, reference, or meaning. In this conclusion, analysis, happily, agrees with traditional usage, intuition, and common sense.

Concluding Considerations

We are, I believe, now prepared to answer definitively the long-vexed question: *What, if any, sort of existence have point spaces of four or more dimensions?*

As we have seen, the conceptual space cS_3 of ordinary geometry is an affair involving extension; it is a triply extended conceptual spread or expanse: three independent linear extensions in it may be chosen; these suffice to determine all the others. So much is as certain as anything can be. It is equally certain that we can, for we do without meeting contradiction, by means of postulates or otherwise, conceive (not perceive or imagine) a quadruply extended spread or expanse, one, that is, in which it is possible to choose four independent linear extensions, and then by reference to these to determine all the rest. There is not the slightest difference in kind among the four independents and not the slightest difference between any three of these and the three of cC_3. The spread or expanse thus set up is a cS_4; like cS_3, it is purely conceptual; the extension it involves is, in kind, identical with that of cS_3; it contains spreads of the type of cS_3 as elements just exactly as a cS_3 contains planes or spreads of type cS_2 as elements; it differs not at all from cS_3 except in being one degree higher in respect of dimensionality. In a word, cS_4 (and, of course, cS_5, and so on) has the same kind of existence as cS_3. It is true that cS_3 is "imitated" by our sensible space sS_3, whilst there is no sS_4 thus imitating cS_4. But this writing is not intended for one who is capable of thinking that the mentioned sensible imitation or imitability of cS_3 confers upon the latter a new or peculiar kind of existence.

But one thing remains to be said, and it is important. If one denies that cS_3 has the conceptually extensional existence, above alleged, then, of course, the denial extends also to cS_4, and the two spaces are, in respect of existence, still on a level. If the denier then asserts, and such is the alternative, that cS_3 is only the ensemble of number triads, etc., as above explained, then, if he be right, cS_4 is only, but equally, the ensemble of ordered quatrains, etc., of numbers. Here, again, cS_3 and cS_4 have precisely the same kind of existence. The conclusion is that *hyperspaces have every kind of existence that may be warrantably attributed to the space of ordinary geometry.*

Cassius J. Keyser.

Columbia University.

DISCUSSION

THE PROBLEM OF VALUES[1]

ALL members of the Philosophical Association owe a debt of gratitude to the executive committee for formulating the question to be discussed at the next meeting. I am aware of no better way of expressing my own gratitude than to comply promptly with the request of the committee for submission of additional formulations of the question.

I shall first make a few remarks upon the formulation by four members of the committee.[2] I assume that in the question, "Is value something which is ultimate and which attaches itself to 'things' independently of consciousness, or of an organic being with desires and aversions?" the "or" is to be understood as marking a genuine alternative between "consciousness" and an "organic being with desires and aversions," not as indicating that the latter clause is in apposition with consciousness or explanatory of it. The alternative is genuine and important: for some may be inclined to connect the existence of values with organic behavior and yet not be willing to equate desires and aversions with "consciousness" —in fact, they may go so far as to hold that "consciousness" (in whatever sense the term is here used) is itself dependent upon matters connected with the desires and aversions of an organic being. Since, however, unconscious desires and aversions may appear to some to involve a contradiction in language, it would seem better to substitute a more objective term, such as selections and rejections; or better yet, to generalize the matter and make the alternative in question to be simply that of connection with the *behavior* of organic beings.

When the question is thus understood, some doubt arises as to the force of the term "ultimate" in the first alternative. Are values if regarded as variables of organic behavior less ultimate than if regarded as things irrespective of connection with organic behavior? And if the answer should be in the affirmative, what is the ground upon which this answer rests?

I have no doubt a successful discussion may be had on the basis of the formulation already presented. In some respects, however, the formulation seems unnecessarily tied up with the idealistic-realistic controversy. I recognize that this complication has the advantage of preserving continuity in the discussions from year to

[1] This paper is presented in response to the request of the committee that further formulations of this problem be submitted for publication.

[2] This JOURNAL, Vol. X., page 168.

year; yet it is possible that the questions at issue might, in the present juncture, be dealt with in the end more effectively if approached by a flank movement. At all events, I venture to submit the following list of questions:

1. Can the question of the status of values in philosophic discussion be approached apart from the question of the status of qualities?

2. Can values be separated from traits of organic behavior? If organic behavior has its own distinguishing traits, does the affirmation that values are traits of organic behavior imply their "subjectivity"? If so, in what sense? Does connection with organic behavior imply their dependence upon awareness?

3. Do values antecede, or do they depend upon valuation—understanding by valuation a process of reflective estimation or judgment?

4. If they antecede, does valuation merely bring them to light without change, or does it modify antecedent values? Does it produce new values? If the latter occur, are the modification and production merely incidental or are they essential?

5. Can the place of intelligence in behavior in general (and in moral conduct in particular) be understood without implying that reflection reorganizes antecedent natural values?

6. What is the meaning of appreciation? Is it a particular mode of apprehending (knowing) values, or is it a name for the direct presence of values in experience? How is it related to valuation and criticism?

7. Does the presence of values in experience in general (or say religious values in particular) have an evidential import? That is to say, does the existence of religious values, for example, prove the existence of any class of objects beyond the values themselves? Or, again, does the presence in experience of any type of values purport to make the mind aware of something in the environment, taking that word in its widest possible sense? [This question may profitably be considered in connection with the first question, regarding qualities.]

8. If the answers to these questions should be in the negative, is the significance of such values for experience and for philosophy thereby determined to be null or illusory? Can an affirmative answer to this question be maintained except on the assumption that all experience is, *ipso facto,* intended to be an awareness of objects?

JOHN DEWEY.

COLUMBIA UNIVERSITY.

SOCIETIES

THE NEW YORK BRANCH OF THE AMERICAN PSYCHOLOGICAL ASSOCIATION

THE New York Branch of the American Psychological Association met in conjunction with the Section of Anthropology of the New York Academy of Sciences on November 25 and on February 24. The following papers were presented at these meetings:

Difference Tones and Consonance:[1] F. KRUEGER.

The Attempt to Measure Mental Work as a Psycho-Dynamic Process:[2] RAYMOND DODGE.

The Psychology of the Earthworm: ROBERT M. YERKES.

This is a preliminary report of an investigation, now in progress, the purpose of which is (*a*) to demonstrate whatever ability the earthworm may have to acquire habits of a certain order; (*b*) to discover the characteristics of any habits which appear; (*c*) to enumerate and evaluate the various external and internal influences on habit-formation; (*d*) to ascertain the degree of permanency of the habits, and (*e*) to discover their relations to the anterior ganglia (brain).

By means of a T-shaped maze constructed from plate glass, specimens of the manure worm, *Allolobophora fœtida,* were tested. The maze was placed with the stem directed toward the light. Across one of the arms a piece of sandpaper was placed and, just beyond it, a pair of electrodes. The other arm was left open so that the worm might escape to an artificial burrow. The worms were driven into the T by light and the chief motive for escape therefrom was the tendency to avoid light. It was the purpose of the test to demonstrate (*a*) any ability which the manure worm may possess to acquire a direction habit and (*b*) to associate the tactual experience of contact with sandpaper with the electrical shock which regularly followed the tactual stimulus in case the worm continued to move forward after reaching the sandpaper.

Trials were made in daily series varying in number from 5 to 20. The 5 trial series were found, on the whole, more satisfactory.

Referring now exclusively to the results obtained for a single worm which has been under observation since October, 1911, the following results may be presented: (1) *Allolobophora* is capable of acquiring certain definite modes of reaction. (2) Modifications appear as the result of from 20 to 100 experiences. (3) The behavior is extremely variable because of variations in external conditions and in

[1] See "Consonance and Dissonance," this JOURNAL, Volume X., page 158.

[2] See *Psychological Review,* January, 1913.

the condition of the worm itself. (4) There is a tendency to follow the mucous path through the apparatus, but this is not sufficiently strong or constant to yield perfect results. (5) The following are the chief modifications which have been noted: (*a*) increased readiness to enter the apparatus and to desert it for the artificial burrow; (*b*) apparent "recognition" of the artificial burrow which is used as "exit tube"; (*c*) a gradual increase in the number of avoidances of the sandpaper and of contact with the electrodes as a result of the "warning" influence of the sandpaper; (*d*) the disappearance of the early tendency to retrace the path through the stem of the T; (*e*) the similar disappearance of the tendency to turn back after progressing well toward the exit tube. (6) The correct performance of a thoroughly ingrained habitual act, of the kind studied in this investigation, is not dependent upon the "brain" (portions of the nervous system carried by the five anterior segments), since the worm reacts appropriately within a few hours after its removal. (7) As the brain regenerates, the worm exhibits increased initiative, its behavior becomes less automatic, more variable. (8) Within four weeks after the operation the regenerated segments appear superficially complete and the worm naturally burrows in a mixture of earth and manure. (9) Two months after the removal of the "brain," during the last four weeks of which period no training was given, the habit had completely disappeared from worm No. 2, the subject to whose responses this paper is devoted, and in its place there appeared a tendency to turn in the opposite direction to that demanded in the training. (10) Systematic training for two weeks resulted in the partial reacquistion of the original direction-habit.

The general results which have just been stated are subject to modification in the light of additional data. To the experimenter it seems that the particular individual which has been longest under observation is in many respects exceptional. It is perfectly clear, however, from results obtained with other individuals that important modifications in behavior appear as the result of training. It is equally certain that direction-habits are not readily acquired.

Psychology as the Behaviorist Views It:[3] JOHN B. WATSON.

Methods of Orientation and Imaginary Maps: C. C. TROWBRIDGE.

The author classified the methods of orientation under two heads. The first was called the *domi-centric method,* used by all living creatures except man in a civilized state. In this case the manner of moving about the surface of the earth relates to a point, usually the home. In the second type, which was called the *ego-centric method,* or cardinal point method, the use is made of the cardinal

[3] See *Psychological Review,* March, 1913.

points of the compass to give orientation, and those points do not necessarily relate to any particular center or home. It is believed that those creatures using a *domi-centric* method have an advantage over civilized man in finding their way home. There may be readily a combination of the two methods in special cases.

In the second part of the paper it was shown that a very large percentage of people, amounting to the order of 50 per cent., are accustomed to *think* of far distant places in an entirely different direction than they really are, amounting to from 45° to 180° from the real direction. The subjects tested *knew* the correct direction within a few degrees. Statistics seem to indicate that individuals having these "imaginary maps" were more apt to be confused with respect to direction than those not having them.

The Probable Explanation of Certain Flock Formations of Birds: C. C. TROWBRIDGE.

This paper also consisted of two parts, and in the first the author showed that birds in a large flock when migrating, in all probability, average their errors with respect to a certain distant destination, and if this is the case the explanation of the migration in large flocks of many species of birds can be explained, also; the principle would prevent single birds from going astray.

The second part of the paper related to the Echelon formation of flight of many large birds when flying in flocks; the explanation given being that it is the most protective arrangement. Evidence was brought forward to show that in this formation the birds in the flock can see forward as well as to the side, these regions are the chief "danger zones" that the flying flock is subjected to. The paper was illustrated by diagrams, and by photographs of blue geese taken by Mr. Herbert K. Job at Marsh Island, on the Mississippi delta.

A Note on the Retention of Practise: F. LYMAN WELLS.

One subject was highly practised in the tapping test 5½ years ago. Six other subjects were highly practised in addition and number-checking tests nearly 3 years ago. The present experiments were made to ascertain the amount and character of the loss during the relative disuse of the functions. In all tests the loss found was about half the percentile amount gained by practise. The renewal of practise does not bring with it an especially rapid practise gain. Persons who gain much in the addition test regularly tend to lose much in it, but this is not true in the number-checking test. Persons who lose much in the one test, however, tend also to lose much in the other, although the amounts of practise gain in them are negatively related.

A Comparative Study of the Illusions and Hallucinations of Dementia Præcox and Manic Depressive Insanity: DARWIN OLIVER LYON.

The various conceptions of the terms *hallucination* and *illusion* were taken up in detail and it was shown that, although no sharp line of demarcation could be drawn between the two terms, yet the distinction was sufficiently fine to warrant their separation in an experiment such as the one under consideration. An hallucination was defined as a subjective sensory image arising without the aid of external stimuli, or, in short, a perception without an object. Illusions were defined as the false interpretation of external objects; *i. e.*, an illusion is the falsification of a real percept. The speaker admitted that cases might occur in which ideas originating wholly in the cortical center might become so vivid as to be taken for sensations that had arisen by stimulation of the sense organs—but he believed that these cases were much less common than is generally supposed.

It was shown that the various authorities differed greatly as to the frequency of hallucinations and illusions in the various forms of insanity. Each of the various psychoses were considered. In *dementia paralytica,* for example, the elder Falret absolutely denied their existence. Kraft-Ebbing says they are so rare that where they are found one should suspect a false diagnosis. Yet Jung, Saury, and Mickle concur in saying that they occur in over one half of all cases.

The part that the various senses play in the fallacious perceptions of the insane was then considered. Though this depends somewhat on the psychosis, both hallucinations and illusions of *hearing* are much more frequent than those of any of the other senses or even combination of the senses. In one form of mania sight hallucinations were found to be greater in number than auditory hallucinations. Hallucinations of taste are very rare. The speaker considered it doubtful if the so-called gustatory hallucinations occasionally seen in *dementia paralytica* were true hallucinations. His experience led him to believe that they were rather the *result* of delusions, in that when a delusion was being "described" by a patient he naturally made his ideas and feelings "fit" accordingly. Of the 361 cases of *dementia præcox* and *manic depressive insanity* tested, only 4 were found having fallacious perceptions of taste, either alone or in combination.

In some cases the patient informs the physician of his own accord regarding his hallucinations and illusions; in others the information sought for must be obtained by some roundabout method. Care must be taken that reported hallucinations are not really illusions;

for example, when in a noisy ward a patient hears herself being called a witch, it is difficult to decide whether she is experiencing an hallucination or an illusion. When, however, the morbid perception occurs in absolute silence we may feel reasonably certain that the patient experiences an hallucination. It was shown that in those cases in which the patient is suspected of endeavoring to conceal the fact that he experiences hallucinations, considerable work may be necessary before their presence or absence can be definitely determined. Careful observation of the patient when he is unaware that he is being watched is, of course, necessary in many cases. Turning the head in a certain direction to listen, gazing at a certain portion of the wall and speaking to it, stuffing the ears with cloth or paper —these and many other "symptoms" lead us to suspect the existence of hallucinations. Evidence of strong emotion, expressions of hate, fear, etc., though not of themselves evidence of hallucinations, warrant further search. The entire test consisted of the following: (1) An examination of the patient's "history." (2) Conversation with the physician and attendants in charge. (3) Various questions and tests varied to suit the case. The question concerning the extent to which we should try to *elicit* hallucinations in an experiment of this nature was taken up in detail.

Tables were then presented showing the results of the tests and conclusions drawn. Of the 173 cases of *dementia præcox* 100, *i. e.*, 58 per cent., had fallacious perceptions; of these, 87 were hallucinations; 8, illusions, and 5 hallucinations and illusions. Of the 188 cases of *manic depressive insanity* 64, *i. e.*, 34 per cent., had fallacious perceptions; of these, only 9 were hallucinations, whereas 51 were illusions. Space does not permit a tabulation of the 18 groups into which the speaker assembled his cases. Suffice it to say that hallucinations and illusions of *hearing* come first—comprising as they do, over one half of all cases. Then come *hearing* combined with *sight,* and then those of *sight* alone. The other senses, either alone or in combination, were but sparsely represented.

H. L. HOLLINGWORTH,
Secretary.

COLUMBIA UNIVERSITY.

REVIEWS AND ABSTRACTS OF LITERATURE

A Short History of Logic. ROBERT ADAMSON. Edited by W. R. SORLEY. Edinburgh and London: William Blackwood and Sons. 1911. Pp. vii + 266.

The philosophical world is greatly indebted to the editor and publishers of this little book for preserving to us in a convenient form and in its entirety the article written by Professor Adamson for the ninth edition of

the "Encyclopedia Britannica." The information which the reader of this review is most likely to desire regarding the new edition is given us in the editor's preface:

"The manuscript of the article has been fortunately preserved—alone among the manuscripts of the author's published writings. It is much fuller than the printed article, a number of passages—some fifty in all—having been struck out by the editor with a view to economy of space. These passages affect both text and notes; they vary in length from a few words to whole sections; they vary also in importance; but the author's own opinion was that the value of his work had suffered by their omission; and with this opinion I agree. In the present book these passages have been restored to their place, so that the article as it left the author's hands is now, for the first time, placed before the reader.

. . . . . .

"It should be borne in mind that the article on Logic was written and published in 1882. The supplementary articles, by which it is followed in this volume, are all contributions to the history of logic; but the first of these—that on the Category, also reprinted from the 'Encyclopedia Britannica,'—dates from six years earlier; and only the last carries the story on towards a more recent development of logical theory. Readers of the author's works do not need to be reminded that his own point of view underwent modification, and that there are some things here which he might have expressed differently had he revised the work himself."

The part of the book which seems to-day especially valuable is the long (58 pages) and admirable chapter on the Aristotelian logic, a chapter which could well be used as an introduction to the Aristotelian philosophy. Besides the article on the Category the other articles included in this book and referred to above by the editor were published originally in *Mind*. Their titles are: "Lotze's Logic," "Lotze's Metaphysics," and "Mr. Bradley's Logic."

Among Professor Adamson's conclusions regarding logic the following seem to be the most general and should be recalled to mind. "Logical theory must of necessity be formal, *i. e.*, abstract and general." It can not consider the specific knowledge of the sciences and include scientific method or the various processes by which the sciences have gained their results. In short, it can not include more than the most general scientific methodology. Again, logic can be identified with the theory of knowledge. "Logical laws, forms, and problems are hardly capable of statement; certainly incapable of satisfactory treatment, except in the most intimate connection with the principles of a theory of knowledge." Hence, of course, the conclusion (a lamentable one for any real progress in formal logic): "It has been imagined that a symbolic logic might be developed which should be independent in all its fundamental axioms of any metaphysical or psychological assumptions; but this is an illusion. No logical method can be developed save from a most definite conception of the essential nature and *modus operandi* of thinking." To which the reply can *in the present day* be given: "But it has been done, such a symbolic logic actually obtains."

WALTER T. MARVIN.
RUTGERS COLLEGE.

The Persistent Problems of Philosophy. An Introduction to Metaphysics through the Study of Modern Systems. MARY WHITON CALKINS. Third revised edition. New York: The Macmillan Company. 1912. Pp. xxvi + 577.

The first edition of this excellent book was reviewed in this JOURNAL in 1907.[1] In 1908 a second edition was already called for. The third, 1912, has given the writer larger opportunity to revise the discussion, and to bring it, in particular to bring the bibliography, down to date.

The principal changes in the second edition concerned a more accurate interpretation of the meaning of causality in connection with the discussion of Hume; corrections and alterations in the account of Kant's *Kategorienlehre;* and a restatement of the writer's view of the self in comparison with the theory of a spiritual substance (pp. 408–409).

The third edition carries the same process further. Within the limits of the former pagination, emendations are made alike in the historical interpretations and in the statements of doctrine. In both fields attention is given to the most recent movements of opinion. And it need scarcely be added that everywhere the same conscientious skill is evident which happily characterized the original work. On the side of history, numerous corrections are made in the account of Descartes's theism (pp. 45, 52, 53), with an added summary of the Cartesian philosophy of nature (pp. 42–43); an improved formulation is given of Hobbes's defense of materialism (pp. 62–63); restatements are offered of certain points in Berkeley's system (pp. 122, 130), and a "more spiritualistic" interpretation advanced of Schelling's philosophy of identity (pp. 339–342). On the side of doctrine, the most important changes are a defense of the writer's doctrine of the self against the objections based on the facts of multiple personality (pp. 409–410), and a recasting of parts of the argument on freedom (pp. 429, 449, 451–452).

In almost every case these changes are improvements. They do not amount, however, to essential alterations, for the point of view remains the same. This appears also in the references to current movements which include something of both interpretation and doctrine. Montague's *Energetik* now shares in the criticisms first leveled at Haeckel and Ostwald (pp. 399–400). A fresh section on neo-realism (pp. 402–404) rebuts the objections of the newer school to the idealistic position. Account is taken (p. 441) of the relations of Bergson's conception of time to absolute personalism. But this continuity of doctrine furnishes no ground for regret. On the contrary, friends and critics will unite in congratulating the author on the merited favor which has given occasion for this revision of her work. And they will count themselves fortunate to possess her conclusions articulated into the outline of contemporary discussions.

WESLEYAN UNIVERSITY. A. C. ARMSTRONG.

The Imaginal Reaction to Poetry. JUNE E. DOWNEY. The University of Wyoming, Department of Psychology, Bulletin No. 2. Pp. 56.

Miss Downey presented to her subjects (12 all told) some 110 poetic

[1] Volume IV., page 440.

fragments selected from the works of Blake, Poe, Keats, Shelley, and Swinburne. The subjects were required to report on the imagery roused by each selection and to pass judgment on the affective value of each. The first tests involved visual presentation of the material. Other tests show the result of auditory presentation. Her tables show that, "Although for every reagent visual images were more frequent than any other kind of image, the excess of such images varied greatly from subject to subject." This was true for both kinds of presentation. Many examples were found of auditory, olfactory, tactual, temperature, pain, organic, kinesthetic, and optical-kinesthetic images, and some few of gustatory images. The degree of conscious control over imagery varies with different subjects. The part played by "inner speech" in the appreciation of poetry is of considerable importance. Motor or auditory-motor inner speech is common, but visual inner speech is very uncommon except in the case of one subject.

An interesting comparison between the five poets comes to light in the author's discussions of "the method of style." Poe, Shelley, and Keats excel Swinburne and Blake in the number of visual images which their lines stimulate. "Poe gives the highest number of successful auditory suggestions; Shelley the highest number of successful olfactory suggestions; Keats the largest number of successful cutaneous images, and Poe the greatest number of successful organic suggestions, with Shelley but slightly behind."

The relation between imagery and the affective judgment is also studied, and it is apparent from the tables that vivid imagery is an important factor in the affective judgment of literature. Visual imagery appears to be the most effective in stimulating pleasant reactions, and organic imagery in unpleasant reactions. Greater consistency appears among the subjects in judging pleasant than unpleasant fragments.

The last division of this study compares affective with esthetic judgments. The general conclusion is that, "although the very pleasant fragment may be esthetic, or the esthetic pleasant, there is no necessary relation of the sort."

Miss Downey's monograph contains an extremely interesting set of records, and her material is presented in such an adequate and straightforward way as to earn the gratitude and admiration of the reader.

KATE GORDON.

LOS ANGELES.

JOURNALS AND NEW BOOKS

REVUE DE PHILOSOPHIE. September and October, 1912. Double number, the first of two devoted to religious experience in Catholicism. The criterion of distinction is that the experiences are within a religion of authority. The first four articles describe the experiences of the great orders. Each order emphasizes a phase of the complete religious

experience which is typical for it and becomes the basis of interpretation for all its experiences. The last four articles treat the philosophy of religion on its theoretical side. *La vie religieuse de l'anachorète, du cénobite et du moine Bénédictin* (pp. 225–256): D. H. QUENTIN. – The goal of anchorite and cenobite is renunciation and personal perfection in a life where all traces of action are banished. To asceticism the monks add obedience as a means to attain their end. The religious tone of the Benedictines is one of deep humility and reverence joined with an internal joy. *L'âme franciscaine* (pp. 257–299): P. UBALD D'ALENÇON. – The Franciscan spirit is one of peace and submission, animated by a passionate personal love for the humanity of Christ. Detachment from the world and extreme poverty are means for the attainment of perfection. *Le Frère Prêcheur* (pp. 300–333): H.-A. MONTAGNE. – The Dominicans show a great variety of experiences. They are animated by the zeal of the apostolic life, and seek in contemplation the truth which they may teach. *Sainte Thérèse et le Carmel* (pp. 334–370): P. MARIE JOSEPH DU SACRÉ COEUR. – The distinguishing feature of the Carmelites is a love of the sacred person of Christ joined with the worship of the Virgin Mary. Love brings about a union with the divine which begins on earth in contemplation. *Quelques réflexions sur la mèthode en psychologie religieuse* (pp. 371–391): J. PACHEN. – Religious psychology has for its field the description and classification of the phenomena of consciousness purporting to be relations with a superior world. Mystics and their writings must be known and scientifically examined. History, sociology, and physiology must aid in the investigation of mystic states. We must ask how we can know God in the mystic state, even though the question can't be answered. Miracles must not be disbelieved, yet the work of religious psychology is to be free from all apologetic motives. *L'épanouissement sociale de l'amour de Dieu* (pp. 392–415): GEORGES GOYAN. – The sacramental life on earth is possible only by association with the love of God, which in its turn expands itself in the loving actions of men. *Sur quelques traits distinctifs de la mystique chrétienne* (pp. 416–488): T. MARÉCHAL. – Man's psychological life is a unity. The various stages in the spiritual life are described. Ecstasy is a synthesis of a negative empirical and a positive transcendental state. Christian mysticism is characterized primarily by the supernatural grace which causes it, and the union of the soul with God which accompanies it. Bibliography. *L'éxpérience, la raison, les normes extérieures dans le catholicisme* (pp. 489–529): HENRY PINARD. – Under the rule of faith reason is to establish truth which the will is to accept as its norm with such loyalty that the soul comes to experience internally a partial union with God.

REVUE PHILOSOPHIQUE. November, 1912. *La perception des grandeurs* (pp. 433–448): B. BOURDON. – Qualitatively different phenomena are brought together under the name of *quantities* and, consequently, we have a habit of neglecting certain real ambiguities in the meaning of

the term. *L'authorité* (pp. 449–464): A. PENJON. – A criticism, not very favorable, of C. B. Huizinga's "Authority," which is said to have made some stir in America. *Le concept de l'idéal* (pp. 465–495): D. DRAGHISCO. – Many problems concerning the nature of the ideal spring from a divorce of discursive reasoning from intuition. *Revue générale. Les revues allemandes de psychologie en 1911:* M. FOUCAULT. *Analyses et comptes rendus.* G. Richard, *La sociologie générale et les lois sociologiques:* G.-L. DUPRAT. Dr. J. R. d'Allonnes, *L'affaiblissement intellectuel chez les déments:* PH. CHASLIN. Dr. G. Saint-Paul, *L'art de parler en public:* DR. CH. BLONDEL. Lévy Bruhl, *Préface à David Hume:* M. SOLOVINE. J. Fabre, *Les Pères de la Revolution:* L. DAURIAC. J. Goldstein, *Wandlungen in der Philosophie der Gegenwart:* M. SOLOVINE. *Notice bibliographique. Revue des périodiques.*

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NOTES AND NEWS

LETTER TO DR. HARALD HOFFDING

February 14, 1913

DR. HARALD HOFFDING,
The University,
Copenhagen, Denmark.

Sir:

We, the undersigned members of the Department of Philosophy in Columbia University in the City of New York, beg to tender you, on the occasion of your seventieth birthday, an expression of warm personal regard and of high appreciation of your long and eminent service as a teacher of philosophy as well as of your valuable contributions to the literature of our subject. Through translations, your writings in the Danish language have been made familiar to students and readers of philosophy in Germany, in France, in England, in the United States, and elsewhere throughout the world. We hope that you may be spared for

many years of additional service to continue to spread abroad knowledge and appreciation of the things of the mind.

We have the honor to be,
Faithfully yours,

HENRI BERGSON,
Visiting French Professor, 1912–13
JOHN DEWEY,
Professor of Philosophy
HERBERT G. LORD,
Professor of Philosophy
DICKINSON S. MILLER,
Professor of Philosophy
WENDELL T. BUSH,
Associate Professor of Philosophy
RUDOLPH EUCKEN,
(Exchange Professor)

NICHOLAS MURRAY BUTLER,
President of Columbia University
F. J. E. WOODBRIDGE,
Johnsonian Professor of Philosophy
FELIX ADLER,
Professor of Social and Political Ethics
ADAM LEROY JONES,
Associate Professor of Philosophy
WILLIAM P. MONTAGUE,
Associate Professor of Philosophy
WALTER B. PITKIN,
Associate Professor of Philosophy

DR. HOFFDING'S REPLY

COPENHAGEN U., April 4, 1913

PRESIDENT BUTLER,

Dear Sir,

It was to me a very great honour and pleasure to receive on the occasion of my seventieth birthday such a splendid message from you and my other philosophical colleagues at Columbia University with the adherence of two so eminent European thinkers as Bergson and Eucken.

I will keep this document among the best remembrances of the day, which was a very beautiful day for me, owing to the warm sympathy with which my colleagues and my students celebrated it.

I remember with pleasure your visit in Copenhagen, and I hope that you and Mrs. Butler are quite well.

I beg you to express my warm and sincere thanks and compliments to the colleagues who subscribed the salutation, and I am, dear Sir,

Your most affectionate
(Signed) HARALD HOFFDING

THE annual meeting of the Southern Society for Philosophy and Psychology was held on April 8 and 9 at the Johns Hopkins University, Baltimore.

DR. WILLIAM ERNEST HOCKING, of Yale University, has been promoted to the grade of professor of philosophy.

PROFESSOR LLOYD MORGAN, F.R.S., has been appointed Herbert Spencer lecturer for 1913 at Oxford University.